Injection Locking of Klystron Oscillators*

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Summary—If certain criteria are met, a microwave oscillator may be synchronized by the injection of a controlling signal into the oscillator cavity. Synchronization is dependent upon oscillator circuit parameters, the ratio of injected power to oscillator power, and frequency difference between the free-running oscillator and the injection signal. Locking has been observed with injection signals 70 db below the oscillator output and 30-db ratios have been demonstrated to be easily realizable. Injection locking may be considered a form of amplification that permits taking advantage of the fact that microwave oscillators are smaller, lighter, less expensive and more efficient than amplifier devices.

The low-frequency theory of Adler is shown to describe accurately the locking phenomena in reflex klystron oscillators and the transient response is extended to determine limitations on the amplification of modulated signals. Experimental verification of the theory is shown for 180° phase modulation of the locking signal at rates up to 100 kc for a VA-201 klystron. Design relations and curves are presented and applications and improvements are discussed.

Introduction

A NUMBER OF investigations have been made into the subject of synchronization of oscillators. Huygens noted that two clocks hung on the same wall tended to synchronize and a paper in German by Moller[1] in 1921 indicates early cognizance of the effect in electronic oscillators. Some recent investigations are described.[2–5] In order for synchronization to be effected, the oscillators must be operating at approximately the same frequency (harmonic and subharmonic relations are also possible) and a mechanism for energy coupling must exist. The synchronization effect may be a blessing or a curse depending on the intended purpose of the equipment.

It is typical in the electronic art that amplifying devices are useful as oscillators at frequencies considerably in excess of their highest useful frequency of amplification. The synchronization of an oscillator by a signal that is a few orders of magnitude below the oscillator power output may be considered a form of amplification. Particularly at microwave frequencies, oscillator devices tend to be smaller, lighter, more efficient and considerably less expensive than amplifier devices. This paper considers some aspects of microwave amplification by the injection locking technique. While the results are similar this technique is to be distinguished from the phase-locked loop type of synchronization which has received much attention of late.[6,7]

Amplification of CW Signals

Adler's[2] theory of synchronization for lower frequency oscillators of the lumped constant circuit type is verified experimentally for the case of X-band reflex oscillators. Considering a conventional oscillator, Alder obtains, subject to certain conditions, the following equation:

$$\frac{d\phi}{dt} = -B\sin\phi + \Delta\omega_0 \tag{1}$$

where

$\Delta\omega_0 = \omega_0 - \omega_1$,
ω = the instantaneous angular frequency of oscillation,
ω_0 = free-running oscillator frequency,
ω_1 = frequency of injected signal,
ϕ = phase difference between the injected signal and oscillator output,
$B = E_1/E \cdot \omega_0/2Q$,
E = oscillator cavity voltage,
E_1 = injection signal voltage,
Q = figure of merit of loaded cavity.

Synchronization of the oscillator to the injected signal is indicated by $d\phi/dt = 0$ (no beat frequency) and (1) becomes

$$\sin\phi = \frac{\Delta\omega_0}{B}\cdot \tag{2}$$

Recognizing that the magnitude of $\sin\phi$ is less than unity, the boundary between the locked and unlocked modes of operation is denoted by a 90° phase shift between E and E_1 and the locking relationship may be

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[1] H. G. Moller, "Uber Storungsfreien Gleichstromenpfang mit den Schwingaudion," *Jahr. fur Draht. Teleg.*, vol. 17, pp. 256–287; April, 1921.

[2] R. Adler, "A study of locking phenomena in oscillators," Proc. IRE, vol. 34, pp. 351–357; June, 1946.

[3] R. D. Huntoon and A. Weiss, "Synchronization of oscillators," Proc. IRE, vol. 35, pp. 1415–1423; December, 1947.

[4] E. E. David, Jr., "Locking Phenomena in Microwave Oscillators," Res. Lab. of Electronics, M.I.T., Cambridge, Mass., Tech. Rept. No. 63; 1948.

[5] T. J. Buchanan, "Frequency spectrum of a pulled oscillator," Proc. IRE, vol. 40, pp. 598–611; August, 1952.

[6] M. W. P. Strandsberg and M. Peter, "Phase stabilization of microwave oscillators," Proc. IRE, vol. 43, pp. 869–873; *ibid.*, vol. 44, p. 696, 1956; *ibid.*, vol. 48, p. 1168, 1960.

[7] W. J. Gruen, "Theory of AFC synchronization," Proc. IRE, vol. 41, pp. 1043–1048; August, 1953.

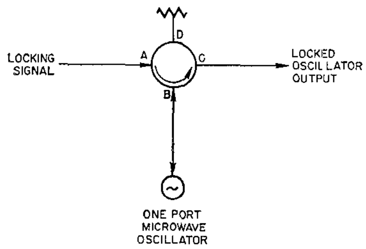


Fig. 1—Oscillator-circulator combination.

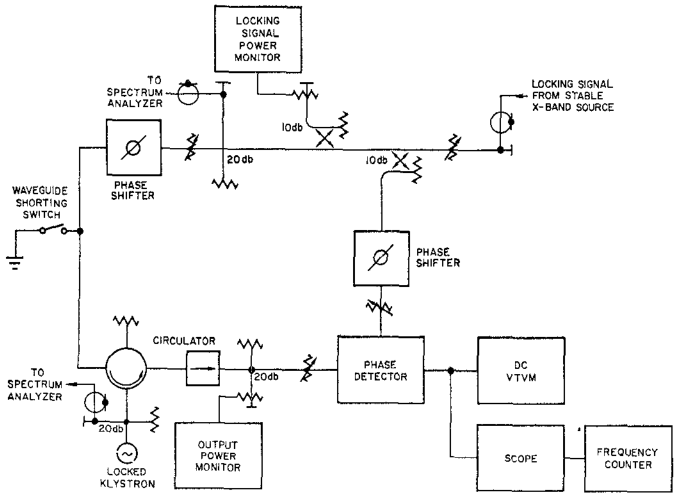


Fig. 2—Injection lock test set up.

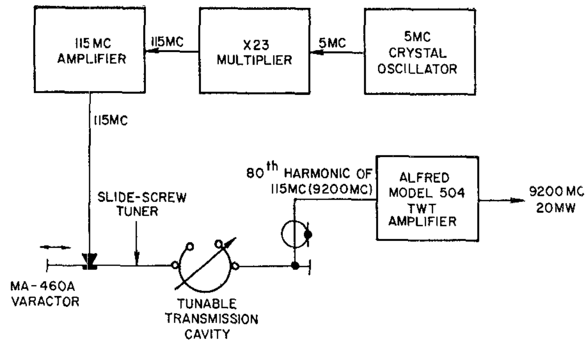


Fig. 3—X-band stable source.

written in terms of cycle frequency and power as

$$\left|\frac{2Q\Delta f_0}{f_0}\left(\frac{P}{P_1}\right)^{1/2}\right| < 1. \tag{3}$$

At microwave frequencies the combination of a one-port oscillator and a circulator makes possible an isolation of the oscillator output from the locking signal input. Fig. 1 illustrates the connection.

Test Set Up

The set up for testing the locked oscillator is indicated in Fig. 2. A stable X-band locking signal is divided in a 10-db directional coupler with the smaller output providing the reference signal for a microwave phase detector. The main injection signal enters the input port of the circulator after passing through a calibrated attenuator, phase shifter and shorting switch. The shorting switch permits convenient removal of the injection signal for measurement of the unlocked parameters. The output of the locked oscillator is attenuated and applied to the signal arm of the phase detector.

The microwave phase detector consists of a folded hybrid tee with a parallel connected matched-reverse polarity pair of diodes. When the oscillator is locked to the injection signal the VTVM gives an indication of the relative phase shift between the injection signal and the locked output; calibration is provided by the precision phase shifter in the reference channel. When the oscillator is not locked to the injection signal the phase detector serves as a mixer for the two input signals, and the frequency difference is recorded by a counter.

The stable reference signal was obtained by frequency multiplication of a 5-Mc crystal controlled signal as shown in Fig. 3. The multiplied signal is applied to a varactor in a waveguide mount and the harmonic at 9200 Mc is selected by a tunable transmission cavity. The TWT amplifier boosts the selected harmonic to a 20-mw power level.

Verification of the Locking Relation

Measurements made of the locking characteristics of several X-band reflex klystrons provided verification of the locking relation; the data presented here are for a Varian VA-201 klystron, serial A773. The procedure is to adjust the frequency of the unlocked klystron to the same frequency as the locking signal, taking care that the klystron is operating at the center of one of the repeller modes. The locking signal is then injected into the oscillator cavity and adjusted to a desired level expressed in db below the oscillator output level. This db figure is termed the injection ratio. Next, the oscillator is caused to have a different free-running frequency by a change of repeller voltage; due to the locking effect the oscillator is pulled from the new frequency to the frequency of the locking signal. If enough change of repeller voltage is introduced a value will be reached at which the locking signal can no longer control the oscillator and unlocking occurs. Removal of the locking signal and measurement of the frequency difference between the free-running oscillator and the locking signal results in data for Fig. 4. The area above the 90° curve represents combinations of injection ratio and unlocked frequency difference that will cause locking on injection of the locking signal. The other curves bound regions of combinations which result in less than 90° phase shift.

The curves of Fig. 4 are for the 150-v mode with 0 db equal to 115 mw; the curves are identical for the 100-v mode with 0 db equal to 62 mw. The curves show that lock was maintained down to an injection ratio of 70 db. The inherent stability of the oscillator sets a lower limit to the injection signal since frequency excursions must remain within the 90° boundary curves. A 70-db injection ratio corresponds to a peak frequency excursion about the locking signal of ± 12 kc which implies a short term frequency stability of about one part in 10^6. In order to achieve this stability the filament and repeller were supplied from battery sources. The repeller typically has a voltage tuning coefficient of 1 Mc/v or a frequency shift of approximately one part in 10^4/v at X-band.

A more useful set of curves may be obtained by plotting the locked oscillator relation

$$\Delta f_0 = \frac{f_0 \sin \phi}{2Q} \left(\frac{P_1}{P}\right)^{1/2} \tag{4}$$

on log-log coordinates. Fig. 5 is a plot of this relation with steady-state phase shift as a parameter. The experimental points are those for the VA-201 klystron indicating that the locking characteristics of reflex oscillators are accurately explained by Adler's theory. In order to plot these curves an average value of Q was determined from several points of Fig. 4; the average value of Q is 116. These curves conveniently relate injection ratio, unlocked frequency offset and steady-state phase shift and are suitable for design application. To ensure that steady-state phase shift is limited to less than a particular value, the operating point of the locked oscillator must lie above the curve representing that particular phase shift. For example, for a 30-db injection ratio the stability of the locked oscillator must be such that the unlocked frequency deviates not more than 600 kc if the phase shift is to be limited to 30°. This represents an unlocking margin of 60° which may be ensured by a feedback technique to be discussed in a following section.

Transient Response

The transient response of the locked oscillator is of interest as it places a limit on the usable modulation

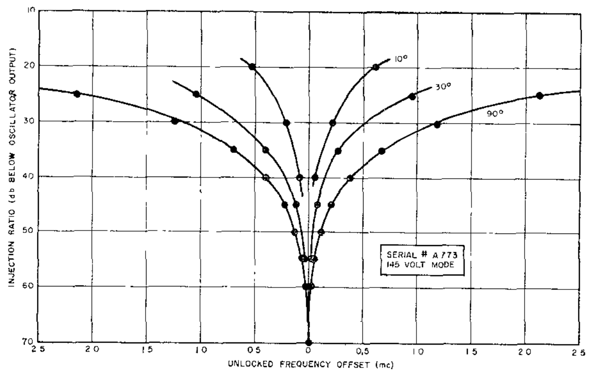


Fig. 4—VA-201B locking characteristics—expanded scale.

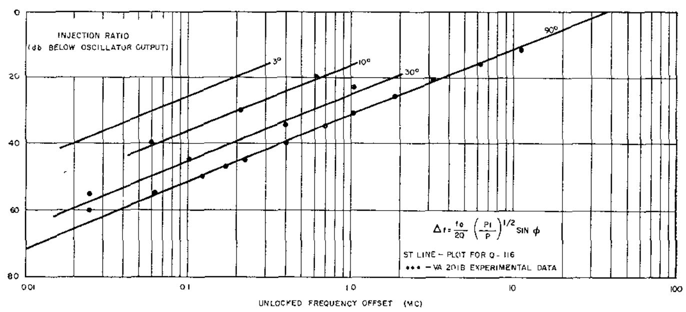


Fig. 5—Experimental verification of the locking relation.

rates of the locking signal. The locked oscillator may be used to amplify signals that are angle modulated if the modulation rate is not too high; amplitude modulation is, of course, not possible because of the limiting action inherent in the oscillator. The transient response is dedetermined by solving (1) for $\phi(t)$ subject to appropriate initial conditions. The solution describes the oscillator response when the locking signal is switched on with an initial phase difference ϕ_0 with respect to the free-running oscillator. It also describes the case of a step function of phase modulation of the locking signal by an amount ϕ_0. The solution of (1) is carried out in the Appendix including the special case of $\Delta\omega_0 = 0$.

For this important special case

$$\phi(t) = 2 \tan^{-1}\left(e^{-Bt} \tan \frac{\phi_0}{2}\right) \tag{5}$$

and Fig. 6 is a plot of this phase response for several values of ϕ_0. The 180° value is of special interest since the phase difference never decreases. This may be made clear by consideration of a damped pendulum analog as suggested by Adler. The response of a pendulum in a viscous fluid is given by (5) where ϕ is the angle between the pendulum and the vertical (Fig. 7). If the viscosity of the fluid is so great that the inertia force is negligible compared to the damping force, the angular speed $d\phi/dt$ is proportional to the restoring force $B \sin \phi$. The 180° phase offset corresponds to the position of unstable equilibrium when the pendulum is vertically upward and the gravity force passes through the pivot point. Any displacement, however small, will cause the pendulum to return to the stable position of vertically downward. Electrically the small displacement is always provided by phase noise either in the oscillator or in the locking signal.

When the locking signal reverses phase the output from the phase detector of Fig. 2 is as shown in Fig. 8. Fig. 8(a) shows the transient response for a 30-db injection ratio with a reference phase such that $\phi = 180°$ in the steady state. When phase reversal occurs the phase detector output becomes maximum in the positive sense and returns to the maximum negative value after the transient period. Returning to the pendulum analogy, the introduction of a small phase transient into the electrical system corresponds to giving the pendulum a slight displacement from its unstable vertical position. The direction of the displacement determines the sense of the angle through which the pendulum returns to the stable vertical position. Fig. 8(b) is a photograph of the phase transient with a reference phase such that $\phi = 90°$ in the steady state. The equal brightness of the positive and negative transients indicates that the oscillator phase is advanced or retarded with equal probability. The density of the transient traces shows that approximately 90 per cent of the reversals have been initiated within 0.3 μsec of the time of reversal of the locking signal for 30-db injection ratio.

The half angle response time can be given the significance of a time constant when comparing different oscillators or injection ratios. The transient response may be written

$$\tan \frac{\phi}{2} = e^{-Bt} \tan \frac{\phi_0}{2} \cdot \tag{6}$$

In particular for $\phi_0 \simeq 180°$ then $T_{1/2} = t(\phi = \phi_0/2) =$ time constant and

$$1 \simeq e^{-BT_{1/2}} \tan \frac{\phi_0}{2} \cdot \tag{7}$$

Solving for $T_{1/2}$ gives

$$T_{1/2} = \frac{\log \tan \frac{\phi_0}{2}}{B \log e} \cdot \tag{8}$$

This expression is only meaningful for $\phi_0 = 180 \pm \Delta\phi$ where $\Delta\phi$ represents a small phase transient. As an example let $\Delta\phi = 3°$ or $\phi_0 = 177°$ and assume an injection ratio of 20 db. Then for the VA-201 klystron $B = 2.38 \times 10^7$ sec^{-1} and $T_{1/2} = 0.152$ μsec. Thus, for the assumed conditions, 0.15 μsec is required for the phase reversal to be reduced to half of its initial value.

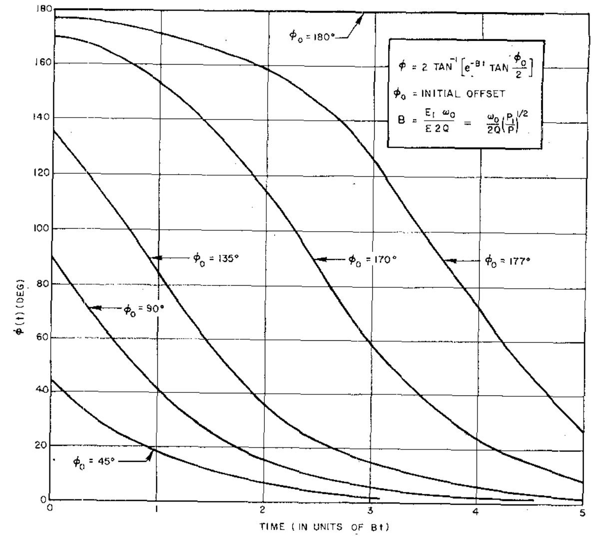


Fig. 6—Locked oscillator transient response for $\phi_\infty = 0$.

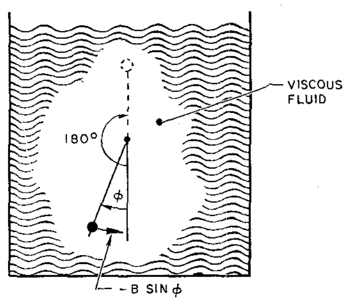
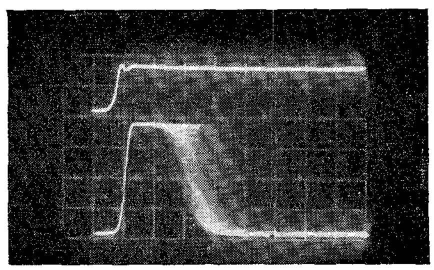
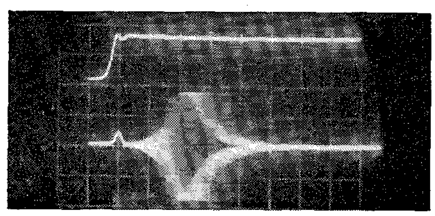


Fig. 7—An overdamped pendulum analog.



(a)



(b)

Fig. 8—Locked oscillator phase transient. (a) Reference phase adjusted for unipolar transient. (b) Reference phase adjusted for bipolar transient. Time scale $= 0.1$ μsec/cm. Top trace in each photo shows the phase reversal of the locking signal.

In general, it may be expected that the free-running oscillator frequency will not be exactly equal to the frequency of the locking signal and a steady-state phase shift will exist through the locked oscillator. A small phase shift is actually desirable if the locking signal is reversing phase as the unstable 180° point is avoided. For example, assume that there is a 3° phase shift through the locked oscillator as shown in Fig. 9. When the locking signal reverses phase to the 180° position the initial phase offset, ϕ_0, is $-177°$ and the oscillator phase returns to the new steady-state position indicated by the dotted phasor. Not only will the jitter be removed from the transient but the return to steady state will always be made in the same direction, *i.e.*, through the smaller angle ϕ_0. Fig. 10 is a photograph of the transient response for a steady-state phase shift through the locked oscillator. Fig. 10(a) shows the response for a 10° phase shift, 10(b) for a 3° phase shift, 10(c) for a 0° phase shift and 10(d) for a 3° phase shift with the reference phase adjusted to show the unidirectional return to steady state.

The general transient solution of (1) with $\Delta\omega_0 \neq 0$ as given in the Appendix is

$$\phi = 2\tan^{-1}\left\{\csc\phi_\infty - (\cot\phi_\infty)\left[\frac{e^{Bt\cos\phi_\infty}\left(\cot\frac{\phi_\infty}{2} - \tan\frac{\phi_0}{2}\right) + \left(\tan\frac{\phi_\infty}{2} - \tan\frac{\phi_0}{2}\right)}{e^{Bt\cos\phi_\infty}\left(\cot\frac{\phi_\infty}{2} - \tan\frac{\phi_0}{2}\right) - \left(\tan\frac{\phi_\infty}{2} - \tan\frac{\phi_0}{2}\right)}\right]\right\}. \tag{9}$$

Fig. 11 is a plot of (9) for $k=0.5$ corresponding to a steady-state phase shift of 30°. When $\Delta\omega_0=0$ the curves of Fig. 6 will be mirrored about the $\phi=0$ line for negative angles, but as $\Delta\omega_0$ increases the curves become increasingly asymmetric as shown in Fig. 11. The angles of unstable equilibrium are $+150°$ and $-210°$.

A comparison between the transient response of Fig. 10(b) and (9) for $\phi_\infty=3°$ and $\phi_0=-177°$ is shown in Fig. 12. Amplitude data from an enlarged oscilloscope photograph is converted to phase information by use of the phase detector characteristic.

Amplification of a Phase Modulated Signal

If the locking signal is reversed in phase by a square wave the locked oscillator will tend to follow the phase variations. If the period of the modulation is long compared to the response time of the locked oscillator the output spectrum will be essentially an amplification of the spectrum of the locking signal. When the modulation period is shorter than, or comparable to, the oscillator response time the oscillator will not follow in phase, but will tend to lock on one of the lines of the input spectrum. The output spectrum is distorted for intermediate values of modulation period.

Fig. 13 compares the spectrum of the locking signal with that of the locked oscillator for a phase reversal of the locking signal every 10 μsec. The output spectrum corresponding to 20-db amplification is a good representation of the locking spectrum. Increasing distortion with increasing gain is indicated by larger amplitudes of the even harmonics and a slight buildup of noise. Fig. 14 compares phase reversals every 10, 5 and 1 μsec for a gain of 20 db.

It may be concluded from observation of the input-output spectra that a ratio of switching time to half angle time constant of 20 or more is necessary for a faithful amplification of the input spectrum for the VA-201 oscillator operating at 20–30 db gain. Sine-wave modulation should be feasible to 500 kc.

Improved Locking Characteristics

In order to lessen the sensitivity of the locked oscillator to relative frequency drifts between the free-running frequency and locking frequency an external feedback loop may be applied in Fig. 15. A signal from the locked output is compared in phase with a fraction of the locking signal. An error signal proportional to phase difference is applied to the klystron repeller after appropriate amplification. The control loop need not be particularly tight or wideband as the principal phase lock is obtained by the RF injection into the oscillator cavity.

Conclusion

The low frequency of Adler has been shown to accurately describe the locking phenomena in microwave reflex oscillators. Injection locking may be considered a form of amplification suitable for CW or angle modulated signals; the transient response of the locked oscillator sets a limit on the usable modulation rate. Experimental results show that a typical reflex oscillator will follow an injected signal 30 db below the oscillator output that is modulated 180° in phase by a 100-kc square waveform. These results may be useful in a situation where space, weight and power considerations make microwave amplifier devices undesirable. This technique is also useful for parametric amplifier pump stabilization.

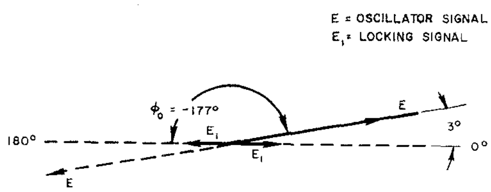


Fig. 9—Locked oscillator phase relationships.

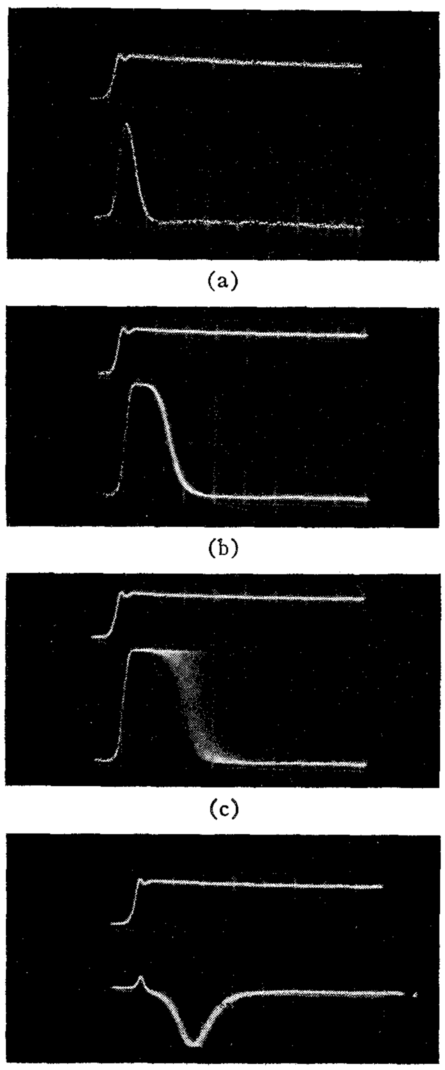


Fig. 10—Locked oscillator phase transient. (a) 10° phase shift. (b) 3° phase shift. (c) 0° phase shift. (d) 3° phase shift. Time scale = 0.1 μsec/cm.

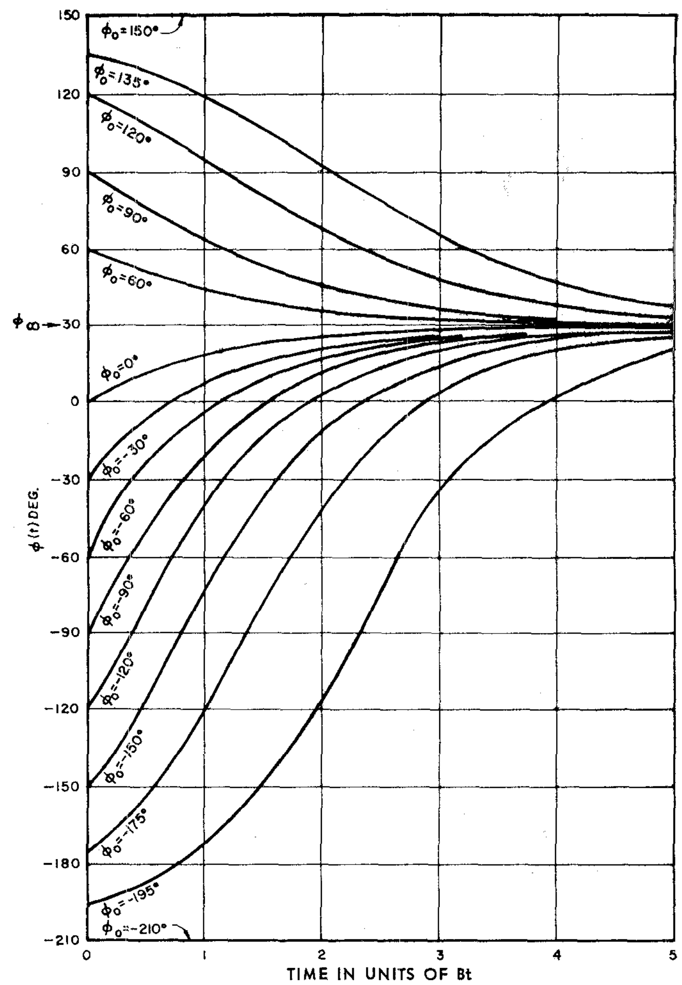


Fig. 11—Locked oscillator transient response for $\phi_\infty = 30°$.

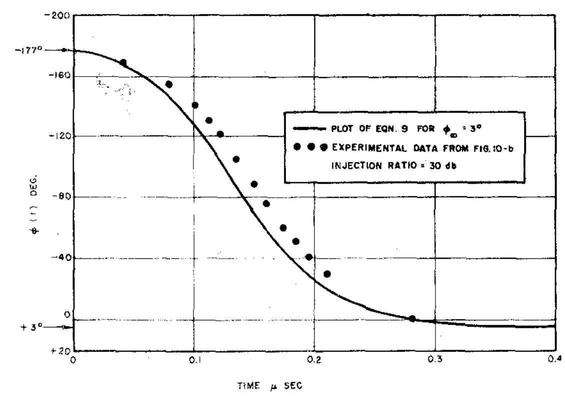
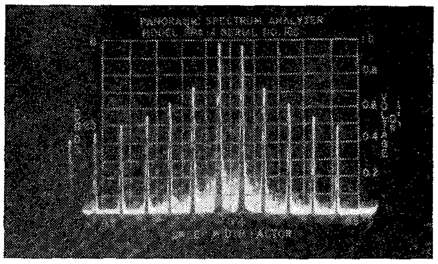
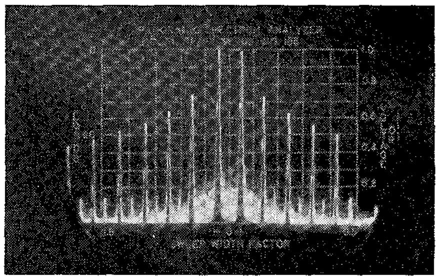


Fig. 12—Experimental and theoretical phase transient comparison.



(a)



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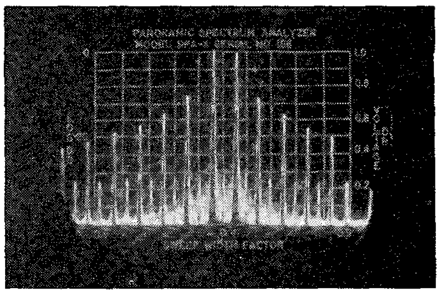

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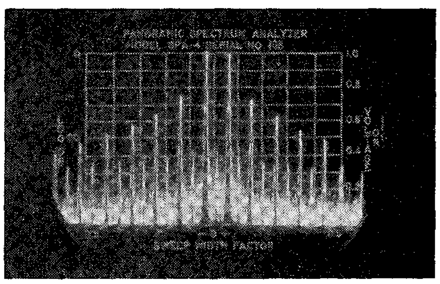
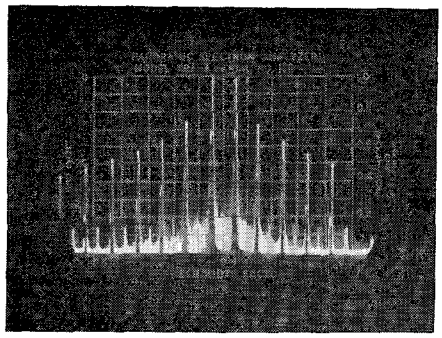
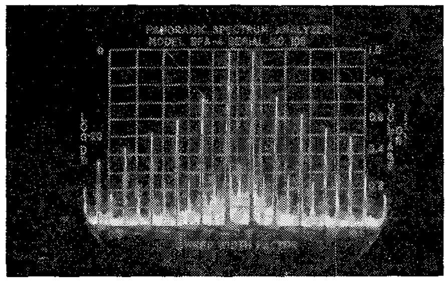


(d)

Fig. 13—Locked oscillator modulation characteristics for 10 μsec phase reversal. (a) Spectrum of locking signal corresponding to 180° phase reversal every 10 μsec. (b) Spectrum of locked oscillator with an injection ratio of 20 db. (c) Spectrum of locked oscillator with an injection ratio of 25 db. (d) Spectrum of locked oscillator with an injection ratio of 30 db.



(a)



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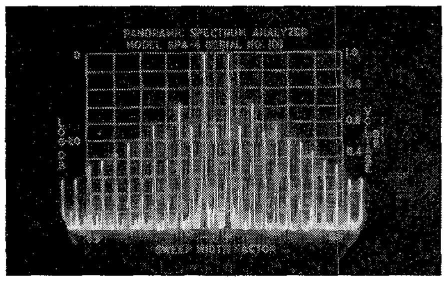

(c)

Fig. 14—Locked oscillator modulation characteristics with a 20-db injection ratio. (a) 10 μsec phase reversal. (b) 5 μsec phase reversal. (c) 1 μsec phase reversal.

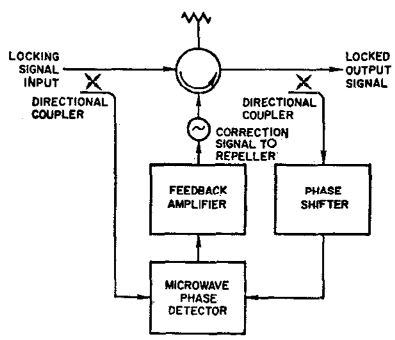

Fig. 15—External control loop to improve synchronization.

Appendix

Transient Response Calculation

The transient response is determined by solving (1) subject to appropriate initial conditions and substitution of $k=\Delta\omega_0/B$. Separation of variables results in

$$\int_{\phi_0}^{\phi} \frac{d\phi}{(\sin\phi) - k} = -B\int_0^t dt. \tag{10}$$

Integration gives

$$\frac{2}{\sqrt{k^2-1}}\left[\tan^{-1}\left\{\frac{1-k\tan\frac{\phi}{2}}{\sqrt{k^2-1}}\right\} - \tan^{-1}\left\{\frac{1-k\tan\frac{\phi_0}{2}}{\sqrt{k^2-1}}\right\}\right] = -Bt. \tag{11}$$

Subject to the condition that synchronization is maintained, *i.e.*, $|k|<1$, (11) may be solved for ϕ in the following form

$$\phi = 2\tan^{-1}\left\{\frac{1}{k} - \frac{\sqrt{1-k^2}}{k}\left[\frac{e^{\sqrt{1-k^2}Bt}\left(\frac{2}{k} - \frac{1-\sqrt{1-k^2}}{k} - \tan\frac{\phi_0}{2}\right) + \left(\frac{1-\sqrt{1-k^2}}{k} - \tan\frac{\phi_0}{2}\right)}{e^{\sqrt{1-k^2}Bt}\left(\frac{2}{k} - \frac{1-\sqrt{1-k^2}}{k} - \tan\frac{\phi_0}{2}\right) - \left(\frac{1-\sqrt{1-k^2}}{k} - \tan\frac{\phi_0}{2}\right)}\right]\right\}. \tag{12}$$

Realization that $k=\sin\phi_\infty$ where ϕ_∞ is the steady state phase shift allows (12) to be written in terms of ϕ_0 and ϕ_∞ as follows:

$$\phi = 2\tan^{-1}\left\{\csc\phi_\infty - (\cot\phi_\infty)\left[\frac{e^{Bt\cos\phi_\infty}\left(\cot\frac{\phi_\infty}{2} - \tan\frac{\phi_0}{2}\right) + \left(\tan\frac{\phi_\infty}{2} - \tan\frac{\phi_0}{2}\right)}{e^{Bt\cos\phi_\infty}\left(\cot\frac{\phi_\infty}{2} - \tan\frac{\phi_0}{2}\right) - \left(\tan\frac{\phi_\infty}{2} - \tan\frac{\phi_0}{2}\right)}\right]\right\}. \tag{13}$$

For the special case of $\Delta\omega_0=0$,

$$\int_{\phi_0}^{\phi} \frac{d\phi}{\sin\phi} = B\int_0^t dt \tag{14}$$

which leads directly to

$$\phi = 2\tan^{-1}\left(e^{-Bt}\tan\frac{\phi_0}{2}\right). \tag{15}$$